AD-A030 888

Application of Graph Theory to Problems in Communications Systems and Networks

Northwestern Univ Evanston Ill Dept of Electrical Engineering

Prepared for

Air Force Office of Scientific Research, Bolling AFB, D C

Sep 76

AFOSR - TR - 76 - 1103

Final Scientific Report

Submitted to

U.S. Air Force Office of Scientific Research

Air Force Systems Command

Grant Entitled

Application of Graph Theory to Problems in Communications Systems and Networks

Research Grant Number

AFOSR-71-2103

September 1976

S. L. Hakimi, Principal Investigator

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR - TR - 76 - 1105	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) APPLICATION OF GRAPH THEORY TO PROBLEMS IN COMMUNICATIONS SYSTEMS AND NETWORKS		5. TYPE OF REPORT & PERIOD COVERED Final 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) S. L. Hakimi		8. CONTRACT OR GRANT NUMBER(s) AFOSR 71-2103
9. PERFORMING ORGANIZATION NAME AND ADDRESS Northwestern University Department of Electrical Engineering Evanston, Illinois 60201		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 9749-06
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332		12. REPORT DATE September 1976 13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Graphs, networks, variable length codes

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research problems successfully considered during the past five years can be broadly classified into five areas:

(1) Communication Networks,
(2) Linear and Non-Linear Circuits,
(3) Variable Length Codes,
(4) Digital Systems and Computer Applications, and
(5) Topics in Pure and Applied Graph Theory.

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A Summary of Progress Made on Grant AFOSR-71-2103

From July 1, 1971 to June 30, 1976

Introduction

The research problems successfully considered during the past five years can be broadly classified into five areas:

(1) Communication Networks,

(2) Linear and Non-Linear Circuits,

(3) Variable Length Codes,

(4) Digital Systems and Computer Applications, and

(5) Topics in Pure and Applied Graph Theory.

We will mention the results obtained in each of these areas. For more elaborate description the reader should consult the annual Interim Scientific Reports for the past five years and of course the papers themselves.

(1) Communication Networks

Two areas in this field were pursued:

(a) Location Theory: This is a field initiated by this author over a decade ago when the notions of p-centers and p-medians were introduced. In [1] the notion of p-medians were generalized. In [2], new techniques were studied for computation of p-center. In [3,4] serious attempts to study the computational complexity of finding p-centers or p-medians of a network were initiated. An invited paper on location theory is presented in [5].

(b) Vulnerability, Survivability and Reliability of Networks: In a major paper [6], the notions of Vulnerability and Survivability of networks are defined and the extent of which these quantities can be controlled in the design of networks is discussed. Reliability of networks is discussed in [7] (this paper has been reprinted in an IEEE Press Book on Large-Scale Networks). Another related problem is the construction of graphs with given connectivity and degree sequence which was attacked in [8]. A tutorial invited paper on these matters was presented in [9].

(2) Linear and Non-Linear Networks

A procedure for synthesizing an n-port specification given by its scattering matrix was given in [10]. This lead to a unified design for RC active circuit implementation of transformers, circulators and gyrators. In [11] a fairly general approach to computer analysis of non-linear circuits was presented. In [12], we give a slight modification of Newton's method for solving a system of non-linear algebraic equations. In [13] a summary of graph theory as applied to problems of interests to circuit theorist is given.

(3) Variable Length Codes

A problem of interest is that of generalization of celebrated McMillan Theorem. This problem involves an examination of the extent of flixability afforded in construction of non-prefix (uniquely decipherable) codes over prefix codes when the cost of transmission of various letters are unequal. These matters are discussed in [14].

(4) Digital Systems and Computer Applications

This area has grown over the past five years to become our biggest effort. Our activities may be divided into two subareas:

(a) Fault Analysis in Digital Systems. In [15], we give a characterization, for the first time, of t-diagnosable systems. This problem was initiated by Preparata, Metze and Chien about a decade ago and has now grown into a field of major interest. In [16] we consider optimal design of t-diagnosable systems. In [17], we give very general theory of characterization of t-diagnosable as well as probabilistically t-diagnosable systems. Also in [17] new results on sequentially t-diagnosable systems are presented. Tutorial papers on these subjects were presented in [18,19]. Finally in [20], we present an efficient algorithm for identifying faults in a t-diagnosable system.

(b) Computer Applications and Algorithms. In [21], we give an algorithm to minimize the use of main memory in series of tree-structured operations. New graph theoretic techniques for file structures and file organizations are explored in [22]. Packing type algorithms are given for finding the number of subgraphs of certain type embedded in a tree [23].

Two problems are actively being pursued. One is a Heuristic Approach to the Cover Problem, and the other involves a Graph Theoretic Approach to Software Validation. Both are very important problems and we have already made good progress on them.

(5) Topics in Pure and Applied Graph Theory

Certain theoretical problems often arise in our attempts to apply Graph Theory and Combinatorics to Systems, Network and Computers. Thus

from time to time we work on problems in graphs whose applications may not be immediate. Such results are compiled here.

The relation between a sequence of integer pairs and graphs are explored in [24]. It is hoped that this may lead to a new theory of degree sequences of graphs. Degree sequences of planar graphs are discussed in [25]. A tutorial paper in the theory of degree sequences is the topic of an invited paper in [26]. A related matter is: given the degree sequence of a graph, when must the graph be pancyclic? This problem had a long standing conjectured solution. In [27], we give a proof of this conjecture.

Problems in distances in graphs and related problems were discussed in [28,29,30].

Upper bounds on the order of a clique of a graph was discussed in [31].

The edge reconstruction problem was discussed in [32]. A number of other papers are pending the referees evaluation and are not discussed here.

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